

Collinear cluster tripartition as sequential binary fission in the $^{235}\text{U}(\text{n}_{\text{th}},\text{f})$ reaction

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Abstract

The mechanism leading to the formation of the observed products of the collinear cluster tripartition is carried out within the framework of the model based on the dinuclear system concept. The yield of fission products is calculated using the statistical model based on the driving potentials for the fissionable system. The minima of potential energy of the decaying system correspond to the charge numbers of the products which are produced with large probabilities in the sequential fission (partial case of the collinear cluster tripartition) of the compound nucleus. The realization of this mechanism supposes the asymmetric fission channel as the first stage of sequential mechanism. It is shown that only the use of the driving potential calculated by the binding energies with the shell correction allows us to explain the yield of the true ternary fission products. The theoretical model is applied to research collinear cluster tripartition in the reaction $^{235}\text{U}(\text{n}_{\text{th}},\text{f})$. Calculations showed that in the first stage of this fission reaction, the isotopes ^{82}Ge and ^{154}Nd are formed with relatively large probabilities and in the second stage of sequential fission of the isotope Nd mainly Ni and Ge are formed. This is in agreement with the yield of the isotope ^{68}Ni which is observed as the product of the collinear cluster tripartition in the experiment.

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I. INTRODUCTION

The observation of two and more nuclear fission products in the fission of ^{235}U with thermal neutrons and in the spontaneous fission of ^{252}Cf has opened a new area of study in the nuclear reactions. This phenomenon is connected with the appearance of cluster states in nuclear reactions and it is the manifestation of the shell structure which is responsible for the production of isotopes with magic numbers of neutrons and protons. When a massive nucleus loses its stability and goes to fission, first of all clusters are formed as future fragments having the neutron or/and proton number nearby the magic numbers 28, 50, 82 and 126. In the case of ternary fission one observes fragments with the charge number 28.

This work is devoted to the study of spontaneous and induced fission of heavy nuclei. There is no full understanding of the fission process and of the dependence of the probability of formation of reaction products on the fission stages. Though one event of the collinear cluster tripartition (hereinafter CCT) process occurs against 1000 events of binary fission, the knowledge of its mechanism gives us a better understanding of the process of spontaneous and induced fission. A theoretical and experimental knowledge of the nature of multicluster fission of various nuclei will promote the construction of a full picture of fission.

The role of the nuclear shell structure in the formation of fission products appears in the observed asymmetric mass distributions depending on the full number of neutrons and the excitation energy of the system undergoing to fission. The other manifestation of the nuclear shell structure occurs at CCT which was investigated by the FOBOS collaboration [1–4]. In this paper, the authors attempt to explain reasons for the yield of the observed fragments in CCT in the $^{235}\text{U}(\text{n}_{\text{th}},\text{f})$ reaction. The cross sections of multicluster fission of the U, Pu and Cf isotopes are less than one percent of the corresponding cross sections of binary fission. So the cross section of CCT is comparable with one of the well-known ternary fission with the emission of an alpha particle. Therefore, theoretical interpretation of these processes is required for a full understanding of the mechanism. The ternary fission fragmentation of ^{252}Cf for all possible third fragments using the recently proposed three-cluster model [5] was studied in Ref.[6]. The authors concluded that the theoretical relative yields imply the emission of the ^{14}C , $^{34,36,38}\text{Si}$, $^{46,48}\text{Ar}$, and $^{48,50}\text{Ca}$ as the most probably third particle in the spontaneous ternary fission of ^{252}Cf .

II. THEORETICAL MODEL

For the description of the formation of mass and charge distributions of fission products of heavy nuclei the concept of the dinuclear system (DNS) [7–9] has been applied. The formation of the DNS is an inevitable stage in fission as in fusion. Indeed, at the descent from the saddle point to the scission point, the fissile nucleus looks like a dumbbell and changes its shape and mass (charge) asymmetry by nucleon exchange on the way to the scission point. The initial stage of fission when the shape of the fissionable nucleus transforms from the compact form into the double nuclear system is not considered here. The charge distribution of the fragments is calculated after the crossing of the saddle point and the formation of DNS. In the case of the DNS with a large neck parameter the DNS model is not acceptable because the DNS concept proposes the smallness of the neck connecting the two interacting nuclei relatively to its whole volume.

The similar conception was used in Ref.[10] to estimate the yield of the ^4He , ^8Be , and ^{10}Be in the ternary fission of ^{252}Cf and ^{56}Ni . The authors of Ref. [10] assumed the ternary fission as a two-step process: the binary system is firstly formed and then the ternary system is formed from it by extracting the light charged particle (^4He or $^{8,10}\text{Be}$) into the region between the two heavy fragments. Then this ternary system decays. According to their concept the ternary system exists together for the short time then it decays while in this work the heavy products undergoes to fission after scission from the light fragment of the previous binary fission of compound nucleus.

CCT can be assumed to be the two-stage fission of the sequential binary fissions. In Fig. 1, a sketch of the sequential mechanism is presented. It is obvious, that such a way of division is possible, when the initial nuclei decay through the asymmetric channel. It should be stressed that the tripartition axes are collinear because the fission axes of both stages are in coincidence: the heavy fragment of the primary fission does not change the momentum direction at its fission into two fragments forming the second and third products of ternary fission. We do not exclude the fluctuation of the fission axis. But this physical quantity is not explored by us. Thus, in the consecutive ternary fission of the compound nucleus first two fragments with asymmetric masses and charges are formed. Then the heavy fragment decays again into two parts.

The mass and charge distributions of fission products of heavy nuclei can be calculated

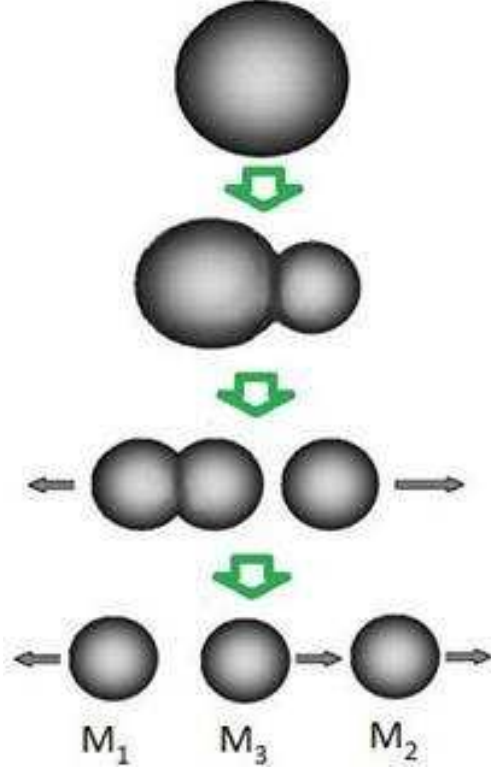


FIG. 1: The presentation of the sequential collinear cluster tripartition of a heavy nucleus.

if the descent from the saddle point up to the scission points continues long enough. During the evolution of the DNS along the mass asymmetry it should not decay into fragments. The DNS concept assumes an interaction between two nuclei by nucleon exchange between them and retaining their shell properties. The validity of the application of the DNS concept to describe deep inelastic transfer and quasifission reactions is evident: we have a DNS in the entrance channel. But its application to explain the yield of products in the fusion-fission, fast fission and spontaneous fission processes requires more thoughts why it is reasonable. With the DNS concept a number of features of deep inelastic transfer reactions, fusion of nucleus, fission, quasifission, fusion-fission and fast fission [9, 11, 12] can be well explained. The main degrees of freedom of the DNS models are the charge Z and mass A asymmetries of the system and the relative distance R between the fragments centers (see Fig. 2). The mass number A of a fragment was found from the minimization of the total potential energy of the fissioning system at the given charge number Z . The DNS model supposes the knowledge of the interaction potential between deformed nuclei with different orientation angles of their axial symmetry axes [9]. Details of the method of calculation of interaction potential is presented in Appendix. This potential as a function of R has a well which allows DNS

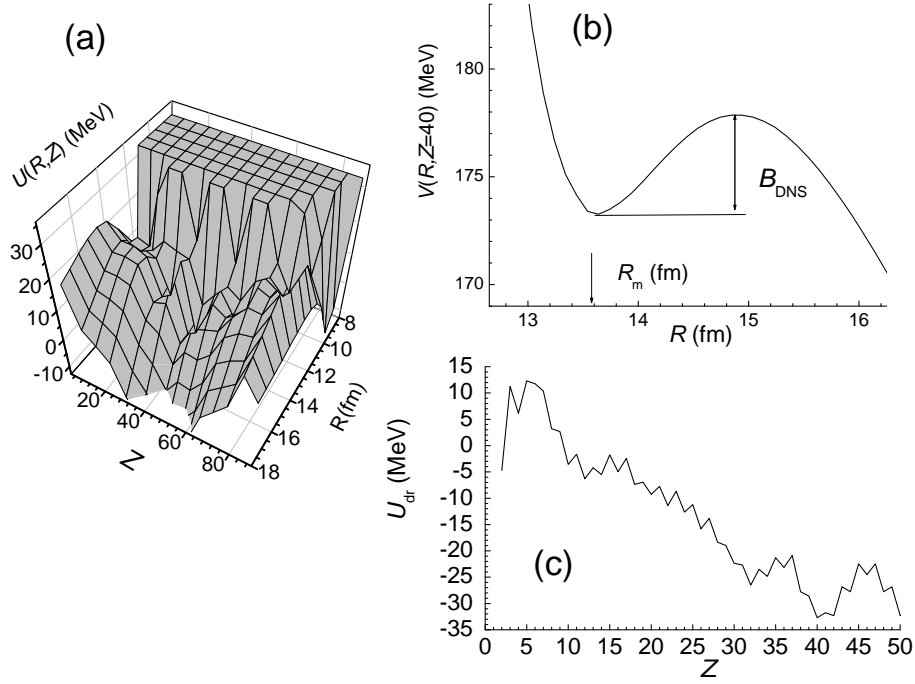


FIG. 2: Potential energy surface for ^{236}U as a function of the relative distance R and a fragment's charge number Z (a); nucleus-nucleus potential $V(R)$ corresponding to interaction of DNS $^{102}\text{Zr}+^{134}\text{Te}$. The depth of the potential well is the barrier B_{DNS} against to decay of DNS (b); driving potential for ^{236}U (c).

nuclei to be in interaction by multinucleon transfer before the decay into two fragments. The depth of this potential well corresponds to the scission point barrier which was discussed in Ref.[13]. This is one of the reasons to explain the fission process. So the delay of scission is connected by the potential well with a definite depth (B_{DNS} , see Fig. 2b).

The delay of the fission process results from the analysis of experimental data showing a competition between particle (neutrons, protons or alpha-particles) emission and fission of fusion-fission reactions. To explain this phenomenon large friction forces or viscosity of nuclear matter have been assumed [14]. It means that the descent from the saddle point up to the scission point occurs by nucleon exchange between already formed nuclei during a long time $((35 \pm 15) \times 10^{-21}\text{s})$ in competition with the neutron emission. During the descent time, mass (charge) equilibrium in DNS can be reached [15] by nucleon exchange which is affected by the driving potential (see Fig. 2c). The information which is used in our model about crossing of the saddle point is the excitation energy generated at the descent from there.

The effects of the relative motion between the fragments (elongation) and mass asymmetry vibration on the fission mass asymmetry were studied in Ref. [16]. The authors concluded that the static potential energy surfaces are quite sufficient to determine the gross features of fission. In Ref. [17], the charge distribution of the reaction fragments with the fixed mass numbers at the spontaneous fission of ^{236}U was calculated by solving the Schroedinger equation with the potential energy of the charge and mass numbers. The authors used the total potential energy which was obtained from the single-particle levels ϵ_i of the asymmetric two-center shell model by renormalizing the sum $\sum_{i=1}^A \epsilon_i$ in the Strutinsky method [18] to the liquid-drop model of Myers and Swiatecki [19] with a modified surface asymmetry constant [20]. The calculated nuclear charge dispersion in the fission of U was in the good agreement with the experimental data for the mass fragmentation chains $A_1 = 141$, $A_2 = 95$ and $A_1 = 142$, $A_2 = 94$.

The DNS model supposes the change of the DNS total energy as change of a sum of the energy reaction balance (Q_{gg} -value) and nucleus-nucleus interaction potential $V(R)$ between its constituents:

$$U(Z, A, R) = V(Z, A, R) + Q_{gg}(Z, A). \quad (1)$$

The Q_{gg} - value represents the change of the internal energy of the system during the reaction: $Q_{gg} = B_1(Z, A) + B_2(Z_{\text{CN}} - Z, A_{\text{CN}} - A) - B_{\text{CN}}(Z_{\text{CN}}, A_{\text{CN}})$, where $Z_{\text{CN}} = Z_1 + Z_2$; B_1 and B_2 (Z_1 and Z_2) are the binding energies (charge numbers) of the DNS constituents. In Fig. 2a the potential energy surface $U(Z, R)$ is presented as a function of the relative distance between the fragments and the charge number Z of a fragment. The nucleus-nucleus potential $V(R)$ shown in Fig. 2b corresponds to the interaction between fragments $^{A_1}Z_1$ and $^{A_2}Z_2$. The charge numbers corresponding to the mass numbers A_1 and A_2 provide minimal values of the potential energy surface.

The driving potential has been found by connecting the minima of the potential well of the nucleus-nucleus interaction (see Fig. 2b) calculated for each charge asymmetry of the DNS, *i.e.* it is the curve lying on the bottom of a valley of the potential energy surface along the Z axis $2 < Z < 90$. If we use $R_m(Z)$ to show the value of R corresponding to the bottom of the potential well for the DNS with the charge asymmetry Z , then the driving potential is defined by using (1) as

$$U_{\text{dr}}(Z, A) = V(Z, A, R_m(Z)) + Q_{gg}(Z, A). \quad (2)$$

The driving potential U_{dr} describing the charge distribution of the DNS fragments formed in the fission of ^{236}U is shown in Fig. 2c. Advantage of such method of calculation of the driving potential for the fissionable nuclear system is the possibility to use the experimental values of binding energy of nuclei B_1 and B_2 [21] that allows us for the account shell effects. The shell effects play the crucial role in the formation of the fission products. The maxima of their neutron and proton numbers are close to the magic numbers which are inherent to the clusters. In our calculations, for the values of binding energy of isotopes which have not been measured yet, we use those obtained from the well known mass tables [22].

The peculiarities of the potential energy surface for DNS allows us to find the basic directions of the evolution and main modes of the decay.

III. APPLICATION OF THE CONCEPT OF THE DNS FOR THE EXPLANATION OF BINARY FISSION

The DNS evolution is determined by the potential energy surface and especially by the driving potential. The driving potential sets the form and position of the maximum of the mass (charge) distribution, as well as the total kinetic energy of the products of the DNS decay. Therefore, the correct description of the experimental data or their interpretation depend on the accuracy of the calculation of the potential energy surface of the DNS [23]. Large probabilities of the formation of fragments with magic numbers 20, 28, 40, 50, 82 and 126 of protons and neutrons (clusters) are obtained by including the quantum shell effects into the consideration. Formation and yields of the fission products with given mass and charge numbers are defined by the landscape of the potential energy surface. We take into account the shell effects to the binding energy of the DNS fragments to explain features of the yields of products in asymmetric fission, which are observed in experiment. The simplest way to take into account shell effects is the use of binding energy values of atomic nuclei from the well known tables [21].

The decay of the DNS may be analyzed as statistical process [10], *i.e.* statistical equilibrium in the charge (mass) distribution of the system can be established in dependence on the given excitation energy and height of the barrier hindering its decay into two fragments. In Ref. [10], the ternary system with a light nucleus between two heavy fragments is assumed to appear from the binary configuration near scission. The theoretical results of the authors

for the charge distributions of the light charged particles which are emitted in spontaneous ternary fission of ^{252}Cf and in induced ternary fission of ^{56}Ni are in a good agreement with the available experimental data.

The charge (mass) distribution of the fission fragments - yield $Y(Z)$ depends on the charge distribution of the DNS fragments $P(Z)$ and decay probability $W(Z)$ of the DNS from the given charge asymmetry state Z :

$$Y(Z) = Y_0 P(Z) W(Z), \quad (3)$$

where Y_0 is a normalizing coefficient for the yield probabilities. The probability of formation of the DNS $P(Z)$ can be found from the condition of a statistical equilibrium as in Ref. [15]:

$$P(Z) = P_0 e^{-U_{\text{dr}}(Z)/T_{\text{DNS}}(Z)} \quad (4)$$

where $T_{\text{DNS}}(Z)$ is the effective temperature of the DNS with the charge asymmetry Z and $U_{\text{dr}}(Z)$ is determined by formula (2). The decay probability of the DNS $W(Z)$ can be found as in Ref. [10]:

$$W(Z) = W_0 e^{-B_{\text{DNS}}(Z)/T_B(Z)}, \quad (5)$$

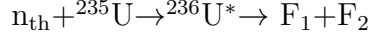
where $T_B(Z)$ is the effective temperature of the system on the barrier (or at the scission point), $B_{\text{DNS}}(Z)$ is the scission barrier for the decay of the DNS (see Fig. 2b). Its value is determined by the depth of the potential well of the nucleus-nucleus interaction.

From the equations (3)-(5) we can see that the yield of the fragments at the decay of the DNS depends strongly on the driving potential $U_{\text{dr}}(Z)$. The equation (4) means that the position of the maxima of the mass distribution corresponds to the minima of driving potential, which is calculated for a given decaying nucleus. Therefore, we consider the minima of the driving potential. For the theoretical research of the yield of fission fragments, which are observed in experiment, the driving potential must be defined sufficiently accurately.

IV. EXPLANATION OF TERNARY FISSION AS CASCADE FISSION IN THE $^{235}\text{U}(\text{N}_{\text{th}}, \text{F})$ REACTION

The knowledge about the driving potential U_{dr} of a fissionable system allows us to make predictions concerning the shape of the mass distribution of fission products. There is the possibility of the explanation of the ternary fission process with specialities of the driving

potential. We have considered the reaction $^{235}\text{U}(n_{\text{th}},f)$ in which CCT was studied in Ref. [2, 3]. According to the sequential mechanism the fission of the heaviest fragment in the first fission stage produces the second and third fragments. Firstly, we shall consider the first binary fission



by the thermal neutron with energy $E_{n_{\text{th}}} = 0.025$ eV. The excitation energy of the compound nucleus $^{236}\text{U}^*$ is 6.54 MeV. The potential energy surface is calculated by formula (1) which includes the shell effects due to the use of realistic binding energies of the interacting nuclei. The energy balances for the different fission modes ($F_1 + F_2$) are presented in Table 1. According to the total energy conservation law the total kinetic energy should not be larger than the energy balance (Q_{gg} - value) in the corresponding channel. Therefore, in the calculation of the potential energy surface for the $^{236}\text{U}^* \rightarrow F_1 + F_2$ reaction the static deformation parameters of the constituents of DNS were used as free parameters to make the barrier of the nucleus-nucleus potential for the exit channel lower than the maximal value of TKE . The driving potential has been calculated with the formula (2) and the result is presented in Table 1. The theoretical value of TKE included the energy carried away by 2 neutrons which are emitted. Therefore, there is a difference between theoretical and experimental values of TKE for the $^{92}\text{Kr} + ^{144}\text{Ba}$ fission channel.

Table 1. Energy balance (Q_{gg}) and theoretical ($TKE(\text{theor})$) and experimental ($TKE(\text{exp})$) values [24] of the total kinetic energy (in MeV) of the fragments for the different fission modes of ^{236}U (the first stage). B_f^H is the fission barrier of the heavy fragment formed in the first stage of the sequential fission. $\beta_2^{(1)}$ and $\beta_2^{(2)}$ are the quadrupole deformation parameters of the first and second fragments, respectively, which correspond to the 2^+ -state and are taken from Ref. [25].

Fission modes	$^{82}\text{Ge} + ^{154}\text{Nd}$	$^{86}\text{Se} + ^{150}\text{Ce}$	$^{92}\text{Kr} + ^{144}\text{Ba}$
Q_{gg} (MeV)	-173.75	-177.976	-193.912
$TKE(\text{theor})$	159.83	168.46	190.69
$TKE(\text{exp})$			170.0 [24]
B_f^H (MeV)	36.33	37.92	39.17
$\beta_2^{(1)}$	0.26	0.19	0.15
$\beta_2^{(2)}$	0.35	0.32	0.19

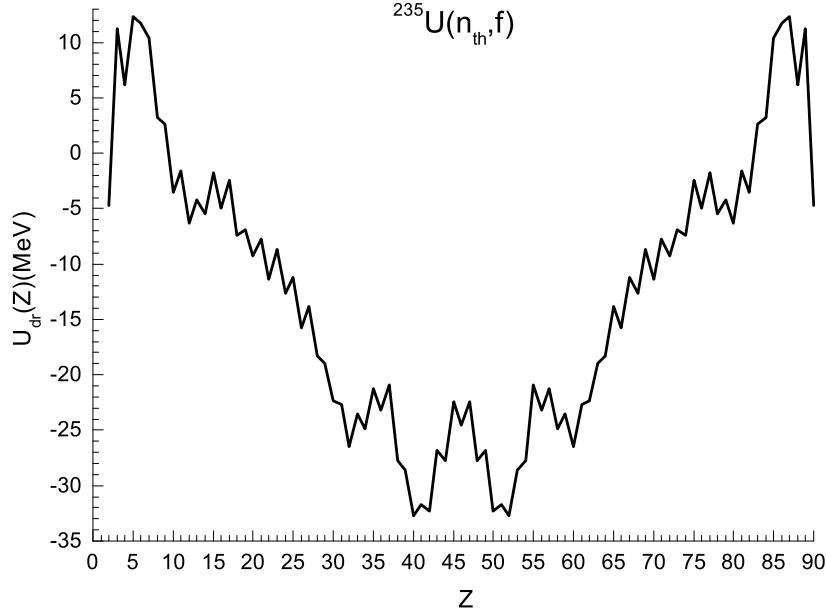
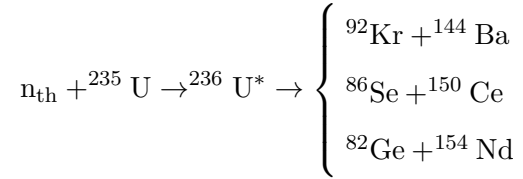


FIG. 3: Driving potential for the $^{235}\text{U}(n_{\text{th}}, f)$ reaction calculated by the use of the binding energies with the shell effects.

As shown in Fig. 4 the yields of the products with the charge numbers $Z=10, 28, 30, 32, 50, 56, 58$ and 60 are well pronounced. In Fig. 3 three minima located at $Z=56, 58, 60$ are responsible for the corresponding maxima at these charge numbers in Fig. 4. In the main fission channels light fragments with the most probable charge numbers $Z=40$ and 42 and the corresponding heavy fragments with $Z=52$ and 54 are produced with larger fission barriers in comparison with the one of the heavier fragments $Z=58$ and 60 (see Table 1). The barrier values are obtained for the ground state of nuclei by the Sierk's model with the rotating liquid-drop model [26]. The shape of fission products is not expected to be in the ground state: as soon as they are prolate deformed, the fission barriers are smaller than presented in Table 1. So, we will consider the fragments Ba, Ce, Nd as fissionable nuclei in the second stage of the CCT.



The results of the calculations for the yields of fragments in the fission of Ba, Ce and Nd are presented in Figs 5, 6 and 7, respectively.

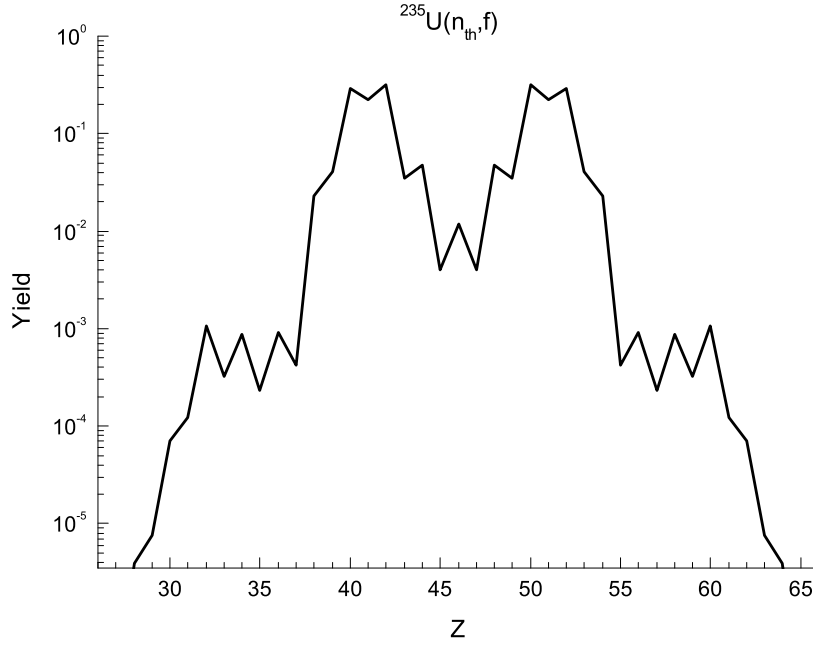


FIG. 4: Yields of the reaction products at fission of ^{236}U calculated with formula (3)

The comparison of the yields of the fission of Ba, Ce, Nd shows that the maxima of the probability of the yields corresponding to the products in CCT $Y(^{144}\text{Ba})=10^{-4} \div 10^{-3}$ in the fission of Ba, $Y(^{150}\text{Ce})=10^{-2} \div 10^{-1}$ in the fission of Ce and $Y(^{154}\text{Nd})=0.1 \div 1$ in the fission of Nd. It means that the formation of the intermediate nucleus Nd and its sequential fission can be considered as the second stage of the ternary sequential fission to interpret the observed yield of the Ge and Ni isotopes [2, 3]. The neutron emission from the fission fragments was calculated as in Ref. [27]:

$$\nu_i = \frac{E_i^*}{B_i^{(n)} + 2T_i} \quad (6)$$

where $B_i^{(n)}$ is the separation energy of the neutron in the fragment i and T_i is the temperature of the fragment. In Figs. 7 and 8, the yields of reaction products for the fission of ^{154}Nd and the corresponding driving potential are presented. The results of the calculations show that ^{82}Ge and ^{154}Nd can emit one and two neutrons, respectively. The fission of ^{154}Nd competes with the emission of two neutrons. The fission probability after neutron emission is smaller than the one before neutron emission because the excitation energy of the fissioning nucleus decreases by neutron emission. In the second stage of ternary fission the nucleus ^{154}Nd fissions into two fragments ^{72}Ni and ^{82}Ge . The maxima of the charge and mass distribution of the second stage fission products lie at these reaction products. Two neutrons are emitted from the ^{72}Ni nucleus: the

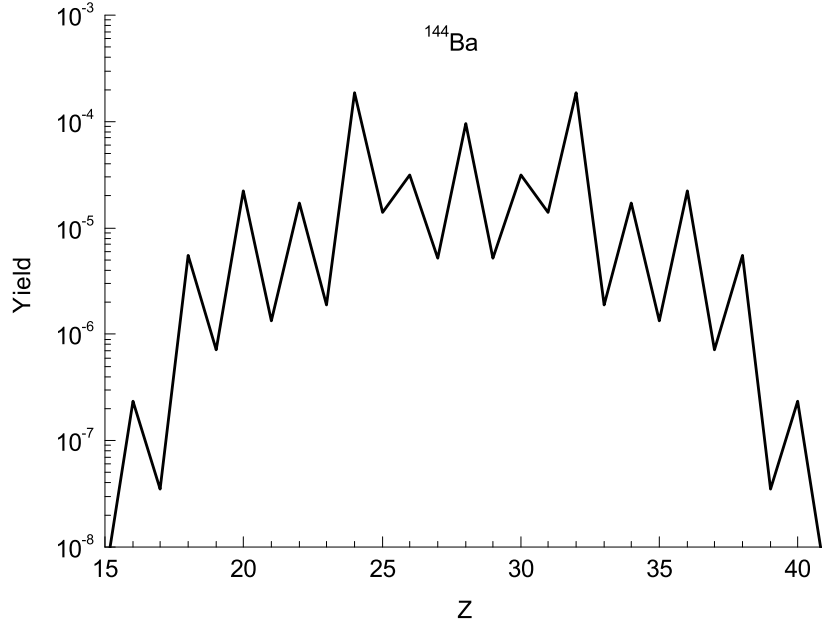


FIG. 5: Yields of the reaction fragments for fission of ^{144}Ba .

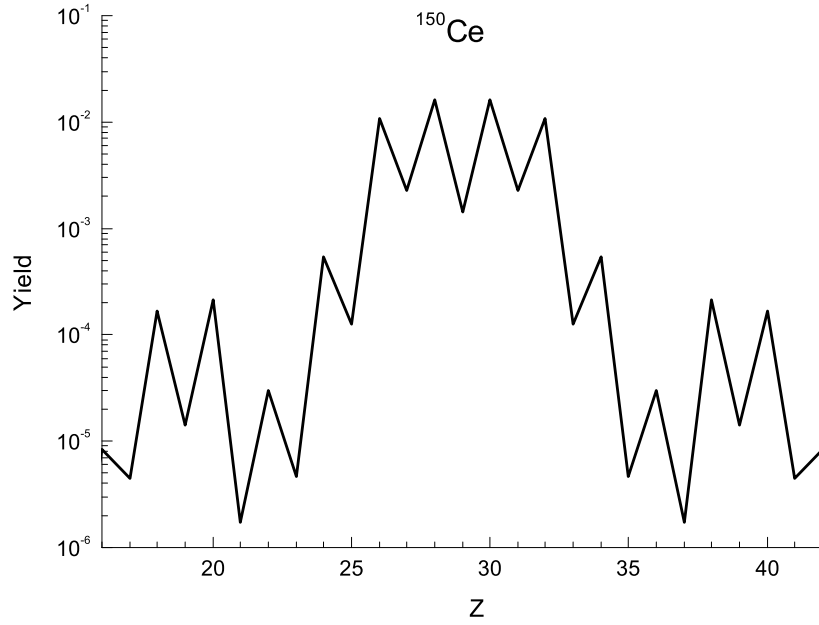


FIG. 6: Yields of the reaction fragments at fission for the ^{150}Ce .

two neutrons separation energy is equal to 10.93 MeV. The similar energy for the two neutron emission from the ^{82}Ge nucleus is larger, namely 12.25 MeV because ^{82}Ge is a double magic nucleus. Thereby, we conclude the following mechanism of CCT of the excited ^{236}U : at the first

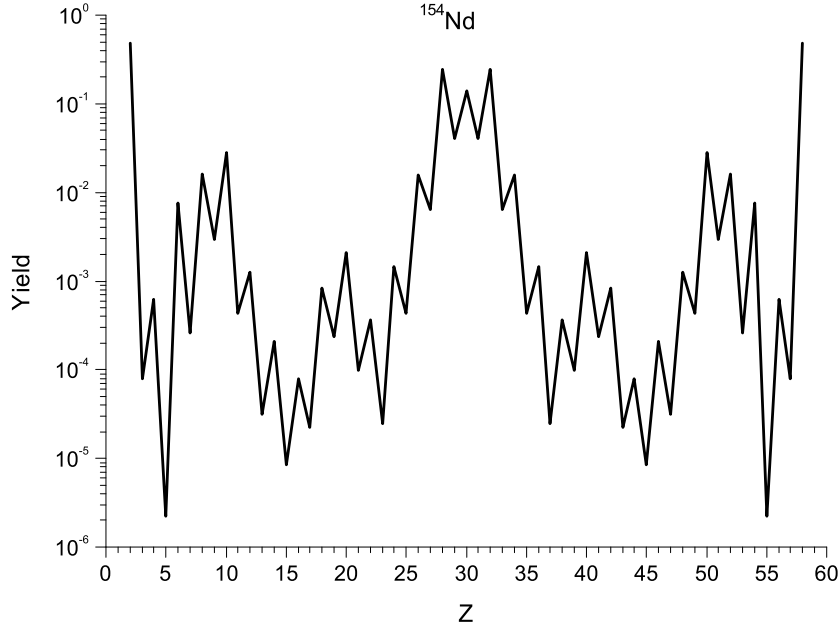


FIG. 7: Yields of the reaction fragments at fission for the ^{154}Nd .

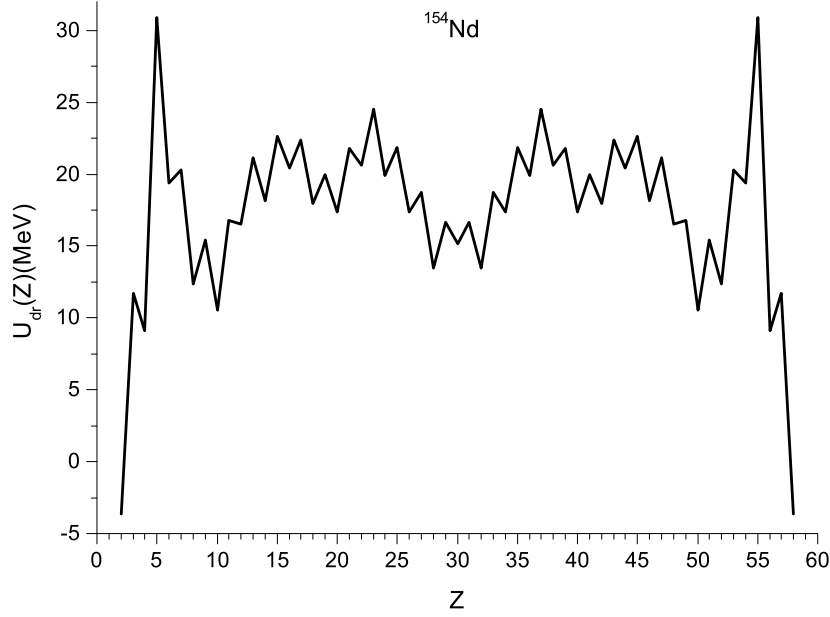


FIG. 8: Driving potential used to calculate the yields of the fission fragments of ^{154}Nd .

stage it decays into the ^{82}Ge and ^{154}Nd products. The excited nucleus Ge emits 1 neutron. In the second stage the ^{154}Nd nucleus decays into two fragments ^{72}Ni and ^{82}Ge , and the excited ^{72}Ni emits 2 neutrons. The charge symmetric fragmentation $^{154}\text{Nd} \rightarrow ^{76}\text{Zn} + ^{78}\text{Zn}$ channel has a

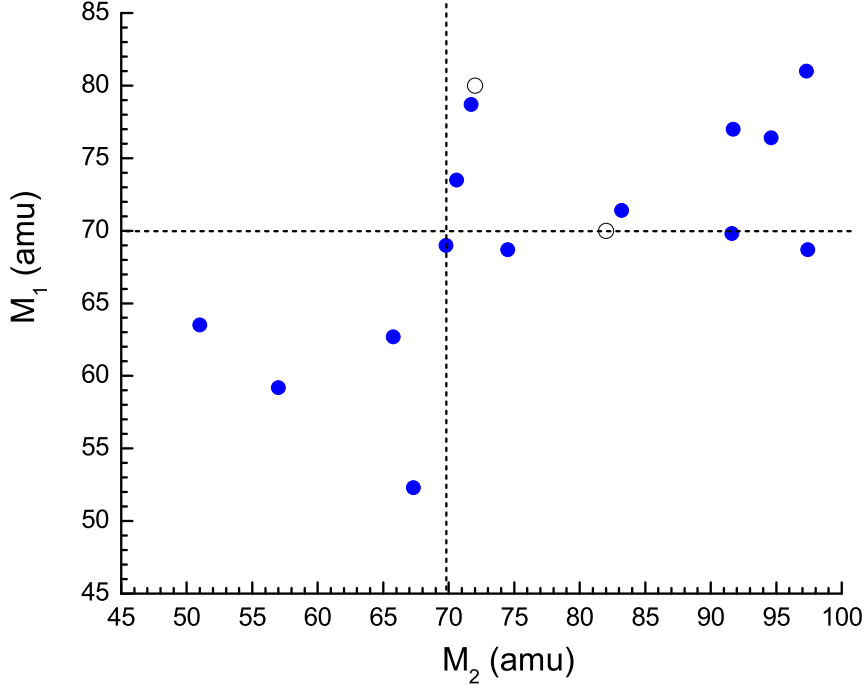


FIG. 9: Comparison of the maximum values of the yield of the tripartition products (open circles) corresponding to the masses $\{M_1 = 80, M_2 = 72\}$ and $\{M_1 = 70, M_2 = 82\}$ which indicate the yield of $^{70,72}\text{Ni}$ and $^{80,82}\text{Ge}$ with the experimental data of the mass-mass distribution (filled circles, from Ref. [3]) of the $^{236}\text{U}(\text{n}_{\text{th}}, \text{f})$ fission fragments registered in coincidence.

smaller probability than the $^{72}\text{Ni} + ^{82}\text{Ge}$ channel. Therefore, our calculations show that the main channel of CCT of the $^{236}\text{U}^*$ is $^{81}\text{Ge} + ^{70}\text{Ni} + ^{82}\text{Ge} + 3\text{n}$. The probability of the ternary fission into this channel is $1.06 \cdot 10^{-3} \times 2.46 \cdot 10^{-1} \approx 3 \cdot 10^{-4}$. The relative probability of CCT to binary fission is approximately 10^{-3} .

In Fig. 9, the maximum values of the mass distributions of the two pair CCT products $\{M_1 = 80, M_2 = 72\}$ and $\{M_1 = 70, M_2 = 82\}$ obtained in this work for the $^{236}\text{U}(\text{n}_{\text{th}}, \text{f})$ reaction are compared with the experimental data of the mass numbers of the two products of tripartition taken from Fig. 6b of Ref. [3]. Authors of Ref. [3] used the “lost fragment” method, when the setup can register only two fragments and the third fragment of tripartition is missed. The experimental data present the massmass distribution of the fission fragments corresponding to the selected events with approximately equal momentum and velocity distributions, as well as with similar nuclear charges. It is seen from this figure that the theoretical yields of the $^{70,72}\text{Ni}$ isotopes accompanied with the yields of the $^{81,82}\text{Ge}$ isotopes are close to the experimental data presented in Ref. [3]. It means

that these experimental events correspond to the sequential two stage mechanism of CCT which is shown in Fig. 1. The analysis of the yields of products having sufficient probabilities in other channels of the first stage fission $^{236}\text{U}^* \rightarrow ^{86}\text{Se} + ^{150}\text{Ce}$ should be investigated and may add new theoretical results.

V. TOTAL KINETIC ENERGY IN FISSION

During the whole process the energy conservation rule must be fulfilled. After the neutron capture we have an excited compound nucleus with the total energy

$$E_{CN}^* + B_{CN} = B_1 + B_2 + E_1^* + E_2^* + V(Z, A, R) + K_1 + K_2, \quad (7)$$

where E_i^* and K_i ($i=1,2$) are the excitation energies and kinetic energies of the constituents of the DNS.

The total potential energy surface is determined according Eq. (1). The total kinetic energy (TKE) and excitation energy of the dinuclear system are determined by the sum of the corresponding energies of the fragments $K_1 + K_2 = TKE$. The total kinetic energy of the fragments is restricted by the value which is found from the total energy conservation law:

$$TKE(Z, A, R) = E_{CN}^* - U(Z, A, R) - E_1^* - E_2^* \quad (8)$$

At large values of R there is no interaction, *i.e.* $V(R \rightarrow 0)$

$$\begin{aligned} TKE(Z, A, R \rightarrow \infty) &= E_{CN}^* - Q_{gg}(Z, A) \\ &- E_1^*(Z, A) - E_2^*(Z, A) \end{aligned} \quad (9)$$

E_{CN}^* is the excitation energy of the compound nucleus. The value of TKE , which includes kinetic energy of the relative motion and surface vibrational energies of the nuclei, depends on the value of $E_1^* + E_2^*$. One can say that the fixed intrinsic energy $E_{CN}^* - Q_{gg}$ is distributed between TKE and $E_1^* + E_2^*$. The latter energy may be spent for emission of nucleons from the system and for its deformation.

For the initial stage, when ^{236}U is formed by the capture of a neutron by ^{235}U , the excitation energy of the compound nucleus is determined from the energy balance of the reaction: $E_{CN}^*(^{236}\text{U}) = E_{n_{th}} + Q_{gg} = E_{n_{th}} + B_1(n) + B_2(^{235}\text{U}) - B_{CN}(^{236}\text{U}) = 6.5 \text{ MeV}$. As we discussed above, in the first fission stage of $^{236}\text{U}^*$ two excited fragments are formed: $^{154}\text{Nd}^*$ and $^{82}\text{Ge}^*$. The excitation energy of $^{154}\text{Nd}^*$ is calculated from the assumption of a full thermodynamic equilibrium

between the formed two fragments $E_{CN}^*(^{154}\text{Nd}) = \frac{A_{Nd}}{A_U}(U_{dr}(BG) - U_{dr}(Z = 60)) = 25.3 \text{ MeV}$, where $U_{dr}(BG)$ is the driving potential at the Businaro-Gallone point of ^{236}U which is equal to 12.3 MeV; $U_{dr}(Z = 60) = -26.5 \text{ MeV}$ is the value of the driving potential for $Z = 60$ (see Fig. 3). We did not calculate the fission probability for $^{154}\text{Nd}^*$ but its excitation energy is enough for fission of $^{154}\text{Nd}^*$. The charge and mass distributions of its fission fragments are determined by the driving potential presented in Fig. 8.

As shown in Table 1., the total kinetic energy of the fragments at the first stage of CCT $TKE(^{82}\text{Ge} + ^{154}\text{Nd})$ is 159.83 MeV, and analogously we can find the total kinetic energy for the second stage $TKE(^{72}\text{Ni} + ^{82}\text{Ge}) = 72.9 \text{ MeV}$.

The driving potential of the $^{235}\text{U}(n_{th}, f)$ reaction has minima corresponding to magic numbers of the protons or neutrons equal to 2, 8, 20, 28, 50, 82, but the decay probability depends on the splitting barrier height $B_{DNS}(Z)$ as a function of the charge number. From this dependence it is clearly seen that the reaction products Ni, Sn and Ge are clusters.

VI. CONCLUSIONS

The sequential fission mechanism of ternary fission of a heavy atomic nucleus has been considered within the framework of the DNS model. Herewith, first the mother nucleus decays into two not alike fragments (asymmetrical fission mode). Then the heavy product decays into two further fragments. The axes of both fission events are in coincidence according to the momentum conservation rule if we assume that the averaged initial angular momentum of the compound nucleus ^{236}U generated by thermal neutrons is very small. The purpose of this work was an estimation of the values of charge and mass numbers of the ternary fission products which are formed with large probabilities and the comparison with the experimental maxima in the charge and mass distributions. For the estimation of these values we calculated the driving potential and yields of fragments of fissionable nuclei. The potential energy is introduced as the sum of the balance energy of the reaction (Q_{gg} -value) and the nuclear interaction potential of the DNS constituents. The use of real binding energies of nuclei, taken from Ref. [21] allows us directly to take into account the shell (quantum) effects in atomic nuclei. Due to the shell effects there is the possibility of the formation of clusters having charge or neutron numbers near the magic numbers 28, 50, 82 in binary and ternary fission of nuclei. The DNS allows to take into account the shell effects in the calculation

of the potential energy surface of the fissioning system. The given method can be applied for the description of the yields of the products of the ternary fission ^{236}U in reactions $^{235}\text{U}(\text{n}_{\text{th}},\text{f})$. As primary channel of the sequential ternary fission we took the channel $^{82}\text{Ge}+^{154}\text{Nd}$. After scission the nucleus ^{82}Ge can evaporate one neutron and it can be registered as the first fission fragment. At the same time, the heavy fragment ^{154}Nd is assumed to undergo to fission into the channel $^{154}\text{Nd}\rightarrow^{72}\text{Ni}+^{82}\text{Ge}$. The neutron emission from the primary fragments ^{72}Ni and ^{82}Ge leads to the formation of the reaction products ^{70}Ni and ^{82}Ge . This result explains the observed yields of the ternary fission products Ni and Ge with a relatively large cross section. Thereby, these estimations within the framework of the DNS model indicate enough well the main channels of CCT. This means, that the chosen theoretical model can be extended for the prediction of quantitative results for the mass distributions of the products of fission.

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Appendix

This method of calculation is applicable at the stage of fission when two fragments are connected with a neck of a small size in comparison to the whole size of the DNS. In this case the interaction between the fragments of the DNS may be described by the diabatic nucleus-nucleus potential for well deformed nuclei with a repulsive core and gives the relaxation of the charge asymmetry degree of freedom.

We use the nucleus-nucleus potential consisting of three parts:

$$V(R, Z, A, l, \beta) = V_{\text{nuc}}(R, A, \beta) + V_C(R, Z, \beta) + V_{\text{rot}}(R, Z, l), \quad (10)$$

where V_{nuc} and V_C are the nuclear and Coulomb parts of the nucleus-nucleus potential, respectively; V_{rot} is the rotational energy of the DNS. In ternary fission of ^{236}U caused by thermal neutrons discussed in this work the rotational energy can be neglected due to the smallness of the partial wave number l . Therefore, the rotation of the DNS is not considered in this work.

The nuclear part V_{nuc} is calculated by the double folding potential:

$$V_{\text{nuc}}(R) = \int \rho_1(r') f_{\text{eff}}[\rho(r, r')] \rho_2(r) d\mathbf{r}, \quad (11)$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{R},$$

where ρ_1 and ρ_2 are the nucleon density distributions of the interacting nuclei; f_{eff} is the effective nucleon-nucleon potential taken from Ref. [28]. The advantage of this Migdal forces is their dependence on the nuclear density of the nuclei:

$$f_{eff}[\rho(r, r')] = C \left[f_{in} + (f_{ex} - f_{in}) \frac{\rho_0 - \rho(r, r')}{\rho_0} \right], \quad (12)$$

where $C=300 \text{ MeV}\cdot\text{fm}^3$, $f_{in}=0.09$ and $f_{ex}=-2.59$ are constants from Ref. [28]. When $\rho(r, r') > \rho_0$ the nuclear part becomes repulsive that corresponds to the appearance of the Pauli blocking principle. For the nucleon density distribution of nuclei we use Fermi functions placed at the center-of-mass of the nuclei which have the distance R between them,

$$\rho_1(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R_1}{a}\right)}, \quad (13)$$

$$\rho_2(|\mathbf{r} - \mathbf{R}|) = \frac{\rho_0}{1 + \exp\left(\frac{|\mathbf{r} - \mathbf{R}| - R_2}{a}\right)} \quad (14)$$

and

$$\rho(r, |\mathbf{r} - \mathbf{R}|) = \rho_1(r) + \rho_2(|\mathbf{r} - \mathbf{R}|). \quad (15)$$

Here, $R_i = r_0 A_i^{1/3} (1 + \beta_2^{(i)} Y_{20})$ is the radius of the i^{th} nucleus, $r_0 = 1.15 \div 1.18 \text{ fm}$, $\rho_0 = 0.17 \text{ fm}^{-3}$, $a = 0.54 \text{ fm}$, and $\beta_2^{(i)}$ are the quadrupole deformation parameters. Orientation angles of the axial symmetry axes of the interacting nuclei are taken as $\Theta_1 = 0^\circ$ and $\Theta_2 = 180^\circ$ in the both stages of the sequential binary fission.

The integral in Eq. (11) is calculated with numerical methods.

The Coulomb potential between the nuclei of the DNS is found by the formula of Wong [29]:

$$\begin{aligned} V_C(R, Z_1, Z_2) = & \frac{Z_1 Z_2 e^2}{R} + \frac{Z_1 Z_2 e^2}{R^3} \sqrt{\frac{9}{20\pi}} \sum_{i=1}^2 R_{0i}^2 \beta_2^{(i)} \\ & + \frac{Z_1 Z_2 e^2}{R^3} \frac{3}{7\pi} \sum_{i=1}^2 R_{0i}^2 (\beta_2^{(i)})^2. \end{aligned} \quad (16)$$

Here, R_{0i} is the radius of the i^{th} spherical nucleus.

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